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ABSTRACT

This position paper is an extension of a presentation delivered to the National Council of Supervisors of Mathematics and the National Council of Teachers of Mathematics at their annual meetings in San Diego, 1978. Four limits in test design are discussed: focus on low-level skills, general adherence to randomness, filling in blanks, and right answers. These limits are suggested as having been transferred to the instructional and learning program in elementary school mathematics. A discussion of areas that might be productive in attacking this problem centers around topics such as children as learners, different kinds of speech, uncertainty and intervention, patterns and language, and memory. (MP)

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THE TYRANNY OF TESTS

in

ELEMENTARY SCHOOL MATHEMATICS

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by
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A Major Address
Presented to the National Council of Supervisors of Mathematics
and the
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THE TYRANNY OF TESTS

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The main thrust of the standard elementary school math program is confined to a narrow range of activities in which progress can be monitored by standardized tests. Limits endemic to the testmakers' art have become generally accepted as the boundaries for textbooks, workbooks, guidelines, continuums -- and classroom strategies and tactics.

Such is the tyranny of the tests.

But this is no polemic against tests. Until education designs and wins public acceptance of more useful instruments for measuring progress, standardized achievement tests will continue as the only -- and therefore necessary -- means for demonstrating accountability to those who pay for and control public education.

The tyranny lies not with the tests themselves. Rather, it exists because educational leadership remains silent -- unwilling or unable to assume responsibility for curriculum development in elementary school math. The math program is shaped in the image of the tests; it is a well orchestrated effort to "teach to the tests."

That characterization has unsavory overtones that may offend ethical and professional sensitivities; it may cause a bit of squirming. But the real indictment of "teaching to the tests" is that it has never worked! . . . . it isn't working now! . . . . and it never will! . . . . not even in terms of elementary school math test scores themselves.

It's a very tight circle ; accountability is measured in terms of those scores. So we put blinders on and drive hard, "teaching to the tests."

We have for some time watched as complex and costly management systems have been created to intensify teaching more directly to the tests. The program hasn't been changed; the effort has been to manage it more systematically. But year after year after frustrating year, tests scores have been resistant, clinging tenaciously at dismal levels. This persistence in failure far transcends any moral or professional laxity implied by "teaching to the tests" . . . . it suggests educational bankruptcy.

SOME ACCEPTED LIMITS IN TEST DESIGN

We have selected four characteristics imposed on testmakers that have been blindly transferred to the basic instructional and learning program in elementary school math:

Limit 1. Tests are able to monitor development of low level computational skills only.

Limit 2. Test items are (and should be) presented in random order.

Limit 3. Tests are pencil and paper activities using a specialized multiple-choice version of filling in blanks.

Limit 4. Tests rely on questions that have a single right answer.

Those limits reflect the present state of the testmakers' expertise. They can be combined and paraphrased into a one-paragraph description of the standard elementary school math curriculum:

> The central concern of the math curriculum is virtually limited to the development of low level skills. The overwhelming bulk of drill and practice presents randomly sequenced paper and pencil activities with children supplying the right answer by filling in blanks, and blanks, and more blanks.

The result of this test-imitating program is that about half of the children by the middle grades have become "I don't get its." Middle-grade teachers describe their deficiencies in everyday language:

> "They can't remember even the basic addition and subtraction facts; they don't understand place value; they can't handle simple word problems . . . . and they don't like arithmetic."

"I don't get its" are human casualties. Few will ever regain any confidence in themselves as learners of mathematics. One can only wonder how education would defend itself if "I don't gets in math" waged a class-action suit as victims of "teaching to the test."

Let's explore the 4 limits listed above that testmakers respect -- limits that have been applied to the math program itself:

Limit 1 - Focus on low level skills

Tests that are generally accepted as reliable are concerned exclusively with distinguishing between those low level skills children have developed and those they have not. They can offer no useful information about progress through the developmental process out of which those skills finally emerge. Nor do they address the important question of whether or not children can and do use those skills as tools necessary in building skills at higher cognitive levels -- as in solving substantial problems involving mathematics.

Consider an outline of a developmental learning strategy that is responsive to the observations of Piaget and the experimental and practical work of Bruner. An essential and unique characteristic of this approach is its careful attention to providing rich language experiences and nurturing language development. The following outline can be applied both to Reading and Mathematics:

A Developmental Learning or Language Development Approach

(1) Begin with doing, seeing and talking -- or more generally, with action, perception and speech -- a dynamic system of behavior that is distinctly human;

(2) Maximize opportunities for children to develop listening and talking skills in a social context;

(3) Solve problems and talk about problems from the outset -- begin to deal with uncertainty;

(4) Learn that talk can be written down using a system of marks on paper;

(5) Notice and discuss regularities, patterns and relationships;

(6) Learn to accept pictures, diagrams and sketches as 2-dimensional representations of the 3-dimensional world;

(7) Learn to make marks on paper as records of verbalized labels -- writing "house" on a picture or "8" to keep track of the outcome of a counting experience;

(8) Read sentences -- understanding what marks on paper are meant to convey;

(9) Write sentences - making marks that convey meaning to others;

(10) Choose to exercise high level cognitive skills, for example

. . . . talk, read and write (language arts)

. . . . solve problems and make investigations (math).

No tests are generally available to monitor progress through the formative steps in this developmental process (listed in the above outline as items 1 through 7); nor do tests purport to have anything to say about solving problems -- item 10 above. Tests address only items 8 and 9. Standardized achievement tests and criterion referenced tests cannot provide useful feedback during important developmental stages; they come in only after children are reading and writing sentences . . . . manipulating symbols in the form of marks on paper.

A math program that focuses on "teaching" the narrow band of skills that tests can monitor has been aptly evaluated by Piaget:

> "The true cause of failure in formal education is that one begins with language instead of beginning with real and material action."

The symbols of arithmetic are shorthand for recording language -- and the referents of that language depend on the past experiences of each child. All meaning is rooted in real and material action.

Primary grade teachers are showing their wisdom when they ask, without expecting any answer, "Why do they make arithmetic become so abstract so soon?"

They deserve an answer: it's because tests operate at the abstract level, and the math program accepts the testmakers' limits!

Limit 2 - General Adherence to Randomness

In everyday life, computational needs do not arise in any order that can be anticipated. The basic facts and processes are encountered in various computational procedures at random. Tests want to provide a reading on how well children can handle bits and pieces as they come up.

Tests would not present these questions side-by-side:

$$\begin{array}{r} 10 \\ +\ 8 \end{array} \qquad \begin{array}{r} 9 \\ +\ 8 \end{array}$$

Testmakers have good and sufficient reasons: some children might not know offhand the answers to 9 + 8 = ___; if it stood as above, children might use their recognition of 10 + 8 as 18 to help them answer a closely related question.

"Randomness" plays an important role in usual testing situations.

But randomness has no useful role in learning situations.

Basically, mathematics is a study of patterns and relationships as in the following closely related examples:

$$\begin{array}{r} 6 \\ +\ 6 \end{array} \quad \begin{array}{r} 6 \\ +\ 7 \end{array} \quad \begin{array}{r} 7 \\ +\ 5 \end{array} \text{ and } \begin{array}{r} 7 \\ +\ 6 \end{array} \quad \begin{array}{r} 5 \\ +\ 7 \end{array}$$

In talking about such groups, children will talk about a "double", a "double and 1 more," a "double and switch 1." They generalize such relationships that reach out from each of the familiar "doubles."

The "switch 1" idea can be used with all examples in which one of the numbers is nine, such as in

$$\begin{array}{r} 9 \\ +\ 4 \end{array} \quad \begin{array}{r} 1 \\ +\ 9 \end{array} \quad \begin{array}{r} 9 \\ +\ 7 \end{array} \quad \begin{array}{r} 3 \\ +\ 9 \end{array}$$

Children learn to see these questions as having the same answer as 10 + 3, 10 + 0, 10 + 6, and 10 + 2.

The idea can be stretched to thinking about "switching 2" when eight is one of the numbers:

$$\begin{array}{r} 8 \\ +\ 2 \end{array} \quad \begin{array}{r} 6 \\ +\ 8 \end{array} \quad \begin{array}{r} 8 \\ +\ 5 \end{array} \quad \begin{array}{r} 3 \\ +\ 8 \end{array}$$

The answers are the same as 10 + 0, 10 + 4, 10 + 3, and 10 + 1.

Nineteen of the basic addition facts have zero as one of the numbers -- so the answer is the other number. Seventeen more have "1" as one of the numbers. Each fact other than the doubles has a "twin" -- if you know that

7 + 5 = 12, then 5 + 7 = 12. These simple patterns or relationships will include all 100 basic addition facts except 3 + 6, 3 + 7, and 4 + 7, and their twins.

The following sequence has no usefulness to standardized achievement test-makers:

$$\begin{array}{rrrrrrl} 7 & 7 & 7 & 14 & 14 & 28 & \\ \times\ 2 & \times\ 4 & \times\ 8 & \times\ 8 & \times\ 16 & \times\ 16 & \text{etc.} \\ \hline 14 & 28 & 56 & 112 & & & \end{array}$$

It is an example of the powerful "doubling" idea in multiplication. In fact, about three-fourths of all 100 basic multiplication facts are involved in this simple doubling relationship.

Tests must avoid considering the "full fact families" as groups:

$$\begin{array}{rrrr} 5 & 9 & 14 & 14 \\ +\ 9 & +\ 5 & -\ 9 & -\ 5 \end{array}$$

and

$$\begin{array}{rrrr} 6 & 4 & & \\ \times\ 4 & \times\ 6 & 4\overline{)24} & 6\overline{)24} \end{array}$$

Three simple ideas can dispose of more than half (51) of all multiplication facts in which 0, 1 or 2 is one of the numbers:

$$0 \times n = 0$$

$$1 \times n = n$$

$$2 \times n = n + n$$

If zero is a factor, the answer is 0; if one is a factor, the answer is the same as the other factors; if two is one of the numbers, it is one of the familiar doubles from addition.

When examples are related, there is something worth talking about; when examples are randomized, there's nothing worth talking about - nothing new to learn.

As one appreciates the crucial role of language experience and language development in learning, randomness must be relegated to the testing mode . . . . and consideration of related examples and situations becomes a requirement in the learning mode.

Workbooks and textbooks are totally committed to randomness - to testing rather than learning. It is understandable that "guidelines" which reflect those texts have nothing much to say about the role of patterns and related examples in learning situations. Such total dedication to King Random leads

classroom teachers to follow that leadership. So the basic curriculum settles down into continual testing, testing, testing - with "learning" left to chance. Some learn anyway - many others don't!

The "randomness" that testmakers must attend to becomes a basic characteristic of the mathematics curriculum. Such tyranny strikes to the heart of the subject - the search for patterns and relationships becomes immersed in a seemingly endless sea of bits and pieces. Mathematics is the least random of all the disciplines.

Limit 3- Filling in Blanks

Testmakers are limited to the "multiple choice" format of "filling in blanks":

$$3 + 4 = ___ \quad (1)\ (8)\ (7)\ (10)\ (N.G.)$$

The child checks the 'right answer" in one of several ways that can be machine-scored. The choice of responses usually includes one or more distractors to minimize the "guess factor."

Such are the constraints and limited options testmakers must accept . . . at least in the primitive state of the art. Textbooks do not use the same format because machine-scoring is not a factor. The textbook and workbook version of "filling in blanks" comes up differently in "consumable" (K-3) and "nonconsumable" materials of instruction. Consumables: fill in the blanks in the book; nonconsumables: copy the examples and fill in the blanks. In no other content area is there so much time spent "filling in blanks."

This format is so pervasive and extensive that many teachers do not look for more useful alternatives.

The sketch below suggests a group of 3 objects and 4 objects:

(a) Let's talk about the idea suggested: such as "3 and 4 more are 7," and "4 and 3 more are 7" and "if you start with 7 and take away 3, there would be 4 left" and "7 take away 4 would leave 3" . . . . "there is 1 more in one pile than in the other," or "if I put one more in the smaller group, they would both be the same size."

(b) We know how to write down the first four ideas which might be called a fact family:

$$\begin{array}{rrrr} 3 & 4 & 7 & 7 \\ +\ 4 & +\ 3 & -\ 4 & -\ 3 \\ \hline 7 & 7 & 3 & 4 \end{array}$$

In the example above, children write full sentences; they are not filling in blanks or copying and then filling in blanks. As they write down that series of sentences, they must also repeat them to themselves as they write - writing and saying the "full sentences" that are the "units of direct memory."

(3) We can also talk about another relationship we noticed - one group has one more in it. The only way generally offered to deal with that is:

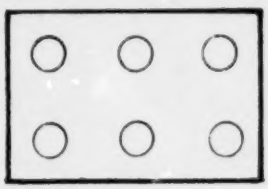
$$3 < 4 \text{ and } 4 > 3$$

but that doesn't indicate "how much" smaller - or larger. Consider:

$$\begin{array}{r} 4 \\ -\ 3 \\ \hline 1 \end{array} \quad \text{and} \quad \begin{array}{r} 3 \\ +\ 1 \\ \hline 4 \end{array}$$

as possible ways to note that idea in the shorthand of arithmetic. (While textbooks never include this "differencing" idea when groups of 4 and 3 objects are suggested, it is actually a very practical way to look at the situation - like shoppers comparing prices .

Here's another sketch:



Think of the circles as cookies - or plums or stones. "If I gave 3 cookies to each of 2 friends, I'd have 6 cookies" and "giving 2 cookies to each of 3 friends would also require 6 cookies." "If I had 6 cookies, I could give 2 to each of 3 friends or 3 to each of 2 friends."

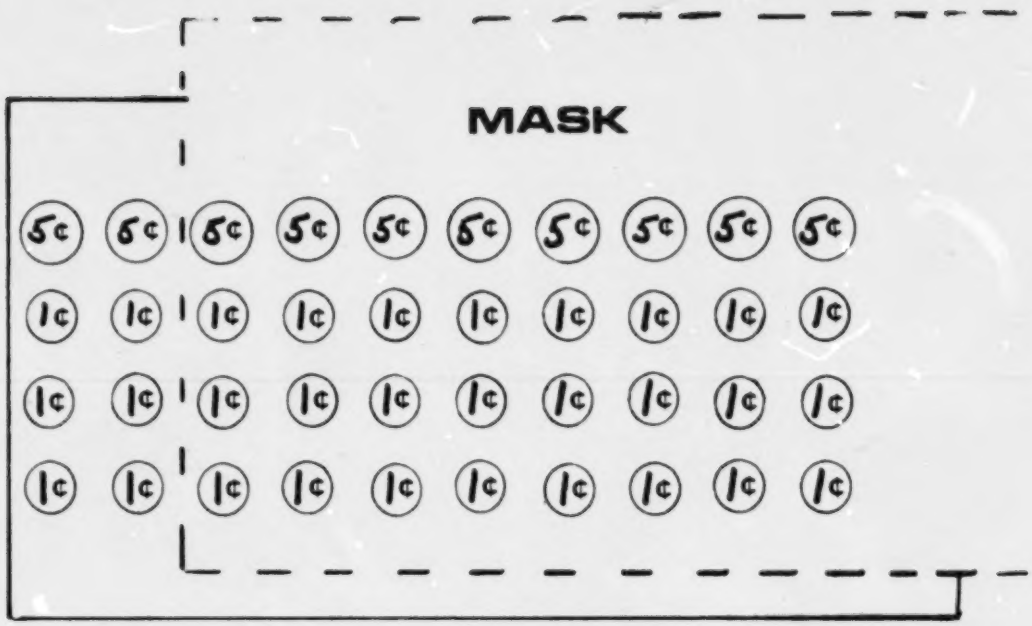
What 4 sentences in the shorthand of arithmetic can we write to capture those ideas?

$$2 \times 3 = 6$$

$$3 \times 2 = 6 \qquad 3\overline{)6}^{2} \qquad 2\overline{)6}^{3}$$

Again, children are not simply "filling in blanks." They are talking about a situation, looking at it in different ways. They are saying and writing complete sentences which are the basic units of memory . . . . and they are talking about the close relationship that ties those "facts" into a tight family group.

Here is a sketch indicating an arrangement of nickels and pennies.



A mask moved from left to right can successively expose 1 more column at a time with 1 nickel and 3 pennies in each column - or 8¢. (We've tried to indicate what would be seen with the mask exposing one column and then a second column.)

"What do you see in the first column?" . . . . a nickel and 3 pennies or 8¢.

"What about 2 columns?" - Now there are 2 nickels and 6 pennies or 16¢.

"How do you see that as 16¢?" . . . ."I see 2 nickels as 10¢ and 6¢ more . . . or I see it as 8¢ + 8¢ and that's 16¢ or I see it as 2 nickels, 5 pennies or another nickel and 1 penny - 15¢ + 1¢ = 16¢."

As the mask is moved to reveal one more column, the discussion changes. Later, the mask can be used again and as each column is exposed, the situation is described as "one 8 is 8; two 8's are 16; three 8's are 24, etc.

Later, children with sketches and masks of their own can write the appropriate arithmetic as suggested by the successive movement of the mask:

8	8	8	8
x 1	x 2	x 3	x 4
8	16	24	32

At no point are there any "blanks" to fill. First, there is a lot of interaction - talking about what you see. As the focus turns to the shorthand of arithmetic - using pencil and paper - children are writing and saying complete sentences.

The test-spawned format of "filling in blanks" should not be transported into classrooms to blot out the rich opportunities there are to stimulate group discussions and to have children saying, hearing, seeing and writing complete sentences.

"Filling in blanks," which testmakers must use, is a tactic that shackles learning when it is transferred into the classroom.

Limit 4- "Right Answers"

Test questions are carefully framed to avoid ambiguity and allow for single right answers. This limit is required by "machine scoring" and provides for "objectivity."

Mathematics is a snap for testmakers. In fact, it is the only subject in which "correct answers" are readily available. A research mathematician once admitted that it gave him uncommon pleasure to say that "7 + 5 = 12" because he knew that "even Sir Isaac Newton couldn't get a better answer."

But behind this accessibility to right answers there are two real dangers: (1) we seldom look deeper than the answer to find out what methods a child used to reach it, and (2) we are lured away from pursuing high level skills - working to solve substantial problems or to understand them better.

Priscilla reached 4th grade testing "at grade level." Her parents were surprised to read a note she brought home: "Can you help Priscilla with addition and subtraction flash cards - she's still counting on her fingers and beginning to fall behind."

The tests had not revealed the fact that she had continued for more than 3 years practicing a method that would be too inefficient at 4th grade. Her teachers had settled for right answers and felt no need to be concerned with her methods.

Consider several children who wrote "12" in the blank in the question 5 + 7 = ___. Child A counted on one hand, then the other and 2 more. Priscilla counted on from 7 and needed only one hand. Child C argued that 5 + 7 was like a double - 6 + 6 - so it must be 12. Child D had a mental image of a domino with 5 spots on one side and 7 on the other; the 7 spots are like a 5 with 2 extra: so it must be 5 + 5 = 10 and 2 more - or 12. Child E looked at 5 + 7 and wrote 12 without any further conscious thought. Behind those "right answers" are methods reaching from most primitive to most sophisticated.

Teachers are continually asking "What's your answer" and seldom asking the more enlightening question "How did you get your answer?"

On page 192 of a second grade textbook the children were asked to "Name 4 ways to make change for a quarter." The exercise seemed so out of place; it was the only question in the 1st and 2nd grade texts of that series that (a) had more than one number for an answer and (2) offered children any choice (there are more than 4 ways).

Comments in the guidebook warned teachers to "beware; children will come up with different answers." No further warnings were needed in that book . . . . two full years with but a single question that would not qualify as a test item.

Mathematics, without searching for answers to less than obvious questions, becomes a caricature of itself and unfit for human consumption.

A math curriculum that confines itself within limits testmakers must accept - all questions must have a single answer - bows to the tyranny of tests.

CHILDREN AS LEARNERS

To focus on children as learners is to put standardized tests in proper perspective -- as the only accepted instruments for measuring final outcomes. Teachers and parents have been weighed down and disheartened by children not learning what they are taught. Regardless of the best efforts of dedicated adults, about half of the children by the middle elementary grades are confirmed "I don't get its." Clouds of frustration hang over children and teachers and parents - and concerned supervisors and administrators.

What a relief it is to turn away from tests - to turn toward finding better and more useful ways to understand children as learners. Frustration gives way as new challenges arise.

The chilly fog of self-doubt recedes as one sees that children are continuously engaged in learning - engaged in action, perception and speech as one and the same dynamic and complex psychological function. Every waking hour (at least) is filled with this never-stopping, ongoing learning process.

The child who seems to drift away, the child who seems to retreat inside, the child who seems impelled to mischief - all are acting, perceiving and talking to themselves if not with others. All are learning, all the time.

What are they doing? What do they see and hear? What do they say to themselves in that inner speech that first appeared as baby babbling and grew into what Piaget calls "egocentric speech" and matures in what we call "talking to ourselves"? There are ways to find at least partial answers to such questions.

What are they learning . . . . and how can we intervene to increase the tempo, the depth and the distance of that learning? How can we influence the direction it takes? There are strategies that can help us move toward these goals.

All of these children come to school as credentialled learners. All come with a command of language demonstrated by three-year-olds asking their parents to "read me the story you readed me yesterday" - certainly not imitating adults, but inventing a word according to a pattern the language often does

follow in forming past tenses. Children perform other prodigious feats of learning. They have become adept at setting goals and using trial and error to reach them - learning to walk upright by age two. They have already demonstrated amazing potentials, and schools have the exciting challenge to nurture that natural learning system, helping all children develop the low level and high level competencies in both the cognitive and affective domains.

If teachers and parents can make even modest headway toward such goals, children will handle "standardized achievement tests" successfully - with ease. I have neither space nor inclination in this paper to describe in detail strategies and tactics to meet the needs suggested. Rather, I would like to indicate with a few broad strokes some main lines of attack that will be productive.

DIFFERENT KINDS OF SPEECH

All animal learning combines action and perception. Human learning becomes far more powerful as speech enters into the learning process.

Very early in their development, children begin "talking to themselves" - using "inner speech." Later they find the utility of learning "communicative speech" in social situations.

Both kinds of speech develop but do not merge. They may reflect each other to some extent and interact, but they fulfill separate functions. Children use their inner speech to organize their view of the world; communicative speech helps them interact with other people.

Mark was in 1st grade and quite able in arithmetic. I asked him to tell how much seven plus nine was. He glanced upward, hesitating ever so slightly before answering, "That's sixteen."

I asked him to tell me how he got that answer. The following comments are transcribed from the tape:

"Nine . . . . I mean . . . . seven take away six . . . . mmm . . . . yah, seven take away six . . . . leaves six . . . . which leaves one into . . . . to give to the ten - and ten and six is sixteen."

We went through several similar examples. In each I asked him to tell me how he thought about it - and made no response to his replies.

Three minutes and 25 seconds into the tape, I asked him again about seven plus nine. He answered "sixteen" and I asked "How did you get that?"

"(Pause . . . sigh!) . . . . 'Cause seven take away one equals six and put that one you took away into the nine which equals ten which leaves the ten and six, and ten and six is sixteen."

When we started, Mark was asked to translate his "inner speech" into "communicative speech" - a new experience for him. What he said at the outset was garbled but offered enough clues to suggest he was talking sense to

himself. But after three and a half minutes of practice, his communicative speech became quite lucid.

In working with other young children, I found their responses to similar discussions to fall into two quite distinct categories: (1) those who do not reveal any use of "inner speech" and (2) those whose use of "inner speech" is obvious, though the communicative speech they use is almost totally garbled at first. Children in the first group are having serious difficulty with arithmetic and dislike it. They accept each question as a test of memory; they often guess wildly or give up immediately. Those in the second group are breezing along and enjoying arithmetic; they have developed an "inner language" they can use to talk their way to transform unfamiliar situations into more familiar and closely related situations.

Children in the first group are rapidly becoming "I don't get its" because they have not developed as part of their "inner language" ways to relate similar related ideas. They have no "inner language" of connections and relationships. They are doomed unless we intervene and help them feel and see and talk about the fact that every situation is part of a bigger pattern. We need to keep asking them how they think about a given situation; we need to tell them how we think about it; we need to let them hear other children talking about the various ways they look at the same condition and talk about it.

The results are dramatic - as if a door had been opened onto a new vista - as if they hadn't realized that those who succeed are always talking to themselves about ways to get from here to there. These children have an "inner speech" deficiency that will be overcome only if teachers and parents are aware of the problem and work patiently and directly to overcome it.

UNCERTAINTY AND INTERVENTION

While the action - perception - speech learning process is continuous, its tempo can vary from "idling away" to "racing uphill at full speed." One effective strategy to increase the tempo of that process is to introduce an element of uncertainty.

Three kindergarteners are asked to sit in chairs in front of the group. They are given sticks with one, two and four balloons.

A prepared tape recorder-player is turned on. "Will one balloon please stand up?" The tape is stopped. After the child with one balloon stands up, the machine is turned back on. "Thank you. Please sit down . . . . Now, will two balloons please stand up." No trouble. "Thank you. You may sit down. Now will three balloons please stand up."

Children look around quickly - considering each bunch. . . but no three! There is some headshaking . . . and "nobody's got three" . . . and voices buzz. Finally someone sees a way out. "One and two stand up - that's three."

The commotion subsides. No difficulty with four balloons. "Five" and "six" present a bit of challenge. "Seven" is finally accomplished with everyone standing.

"Thank you. Please sit down . . . Now, will eight balloons please stand up." The noise level in the group rises and someone calls out "No way!" An observer will notice that children's involvement personally and as a group reflects the degree of uncertainty and puzzlement encountered . . . . and is a reliable indicator of learning on the part of all participants. (As the number of children varies and as the numbers of balloons on each stick vary - sometimes two children with the same number of balloons - the nature of the uncertainty fluctuates.)

Fig. 1 below suggests a common matching activity, connecting numerals and sketches with an appropriate number of spots. A solution is indicated with dotted lines. If you can do it at all, it is a simple minded activity . . . and learning idles along.

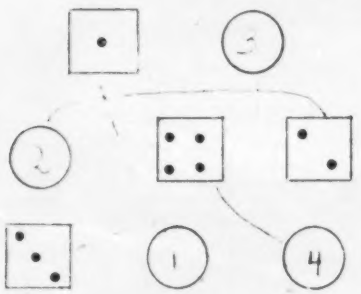


Fig. 1

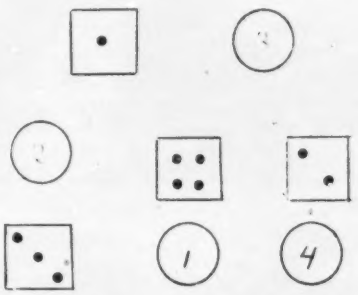


Fig. 2

Fig. 2 is the same arrangement. Lines are to be drawn connecting each numeral with the appropriate number of dots; however, no connecting line should cross any other connecting line - if possible.

Can it be done? In figure 1, lines cross in 4 points. Can you avoid all such crossings?

Most adults, looking at the situation with its limitations, feel the need to plan ahead. Their equilibrium is disturbed. They study the sketch and talk to themselves a lot. They wrinkle their brows and some squint a little. They draw imaginary lines in their heads. Finally, they smile and relax. They see a way to do it. Their equilibrium is reestablished, and the tempo usually slows down. (Some may push on wondering if four numerals and four sketches with dots could be arranged to present an inescapable trap.) Is there a combination of 7 or 8 coins that makes exactly 18¢? Can you cut up a square into

exactly 4 squares, 5 squares, 6 squares and 7 squares? They may or may not be all the same size. A way to cut a square into 4 or 6 squares is shown. What about 5 square pieces and 7 square pieces and 8 square pieces? (There is no trickery involved at all.)

Using only the digits, 1, 4, 5, 7, and 8, how many whole numbers less than 100 can you write that are whole number multiples of 6? (There are not more than 6.)

PATTERNS AND LANGUAGE

All teachers of language arts are aware of the need to draw attention to patterns or regularities in spite of many existing exceptions. The purpose is obvious - children need some guidelines to help them relate new situations to more familiar experiences. The general strategy is to begin with "regularities" and deal with "exceptions" later.

The "silent 'e' principle" is useful. If the word ends in a vowel-consonant combination, the vowel is "short"; if an "e" is added, it is "silent" and changes the vowel to a "long" sound.

Examples	Exceptions
Hat → hat(e)	set → seat
fin → fine	now → known or no
hop → hope	hit → height
etc	etc.

The need to become familiar with patterns leads us to stretch as in "'i' before 'e' except after 'c' or when sounded as 'a' in 'neighbor' or 'weigh'." . . . . and then we haven't taken care of "ancient" or "either" . . . and that's a bit "weird."

What a relief to transfer this search for patterns to mathematics! . . . . where there are no exceptions (except you can't divide by zero). Teachers must be helped to see that the search for patterns is the very nature of the subject - even though those patterns are so reliable! Because regularities are so easy to find does not deter from their significance.

The dictionary of counting numbers begins 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 - and then uses the notion of "place value" so all numbers that follow are combinations of those ten symbols - 10, 11, 12, etc. To read the number 987,654,321, one needs to be able to read any 3-digit number and to recognize that the first comma from the right is a symbol for "thousand" and the second comma stands for "million."

Many "intelligence tests" use numerical sequences because they pose nice problems with which most people are unfamiliar. "Can you fill in the blanks in each sequence?"

1, 3, 5, 7, 9, ___, ___

2, 4, 6, 8, 10, ___, ___

1, 4, 9, 16, 25, ___, ___

1, 2, 4, 8, 16, ___, ___

1, 4, 7, 10, 13, ___, ___

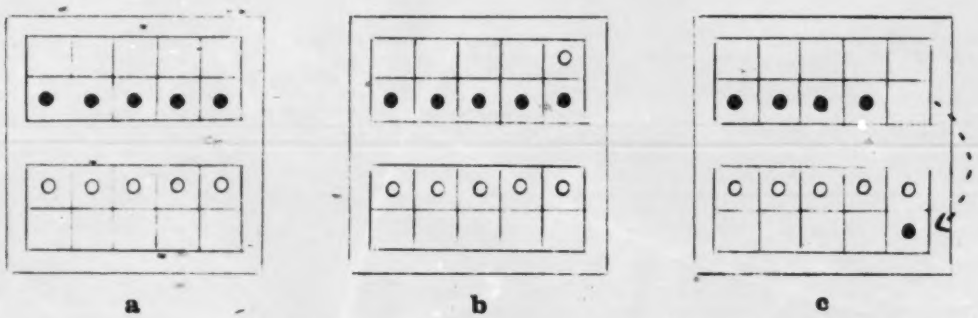
1, 1, 2, 3, 5, ___, ___

3, 7, 11, 15, 19, ___, ___

1, 3, 6, 10, 15, ___, ___

We call the first sequence the "odd numbers" and the second "even numbers": Then the "square numbers" and "the powers of 2" . . . . etc.

How does the investigation of patterns translate into productive classroom strategies? Suppose we arrange counters in pairs of ten-frames as suggested by the sketches below:



a b c

In situation "a" there is a full row of counters in each ten-frame. The counters could all fit in one ten-frame. There are 10 all together - - it suggests a "double" -- 5 + 5 = 10.

How are situations "b" and "c" related to "a"? "There's 1 more in the top frame in 'b'; and 'c' is like 'a' with 1 moved from the top to the bottom frame." "a" is a double; "b" is the same double with 1 more; "c" is the same double with 1 switched.

When one notices relationships, there is "something worth talking about." In this case, we are considering the connection between the familiar doubles and minor variations on those doubles.

Mary went to the store to buy a pencil and an eraser. She found this sign on a special counter:

Sale

Pencils 4¢ Erasers 3¢

Limit - no more than 3 of each to a customer.

It's obvious her purchase will cost 7¢. While she was there, Billy bought some items from the counter and paid 15¢: What did he buy? Remember, the limit - no more than 3 of each to a customer. Can you be sure of what Billy bought?

Maria also made a purchase of one or more from the table. If I know how much she spends can I be sure of the number of pencils and the number of erasers she purchased?

What is the largest purchase that can be made under the terms of this sale? Can you spend exactly 16¢? The clerk could answer any such questions without thinking much about them because on the back of the sign he had prepared a little table:

ERASERS @3¢

PENCILS @ 4¢

	0	1	2	3
0	0¢	3¢	6¢	9¢
1	4¢	7¢	10¢	13¢
2	8¢	11¢	14¢	17¢
3	12¢	15¢	18¢	21¢

Billy bought 1 eraser and 3 pencils. If you know how much Maria spent, you know what she bought. No amounts are repeated. No, you can't spend exactly 16¢!

Patterns and regularities come up in mathematics at every turn: in sequences to be extended, in tying groups of basic facts around those one knows

best, and in "applications" when considered from the clerk's point of view as well as that of the customer.

Pencil and paper are handy to have around when needed, but the true content of mathematics is finding regularities, patterns and connections -- and talking about them. This can be explored by all children at all stages of development.

MEMORY

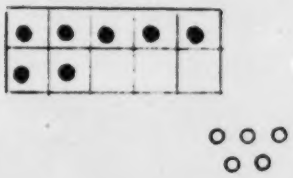
There are two quite different kinds of memory. All animals other than humans are limited to "natural memory." But, speech makes possible a far more powerful strategy we can refer to as "mediated memory."

Natural memory is not necessarily uncomplicated. Children "remember" faces and can pick their parents out of a crowd; children "remember" the characteristics that distinguish cats from dogs . . . . even though adults would find it hard to verbalize those differences. But there is no conscious effort to remember. A person ties a string on a finger as a signal to remember something. On my key ring, there is a key that opens the doors of my car and is also used to activate the starter. It is the only one with a "square top." When I use it to unlock a door, I twist this "top toward the back seat" (it works that way on both doors). When I get to the airport and take a time-slip from the mechanical vendor, I put it "on the visor" so I'll know where it is when I need it to give the attendant when I check out. These are conscious efforts to remember . . . . and they are uniquely human.

Natural memory involves the notion of "Stimulus-Reponse Bonds" S → R . . . . and there is very little anyone can do to help develop this kind of recall. "Mediated memory" is quite different. An aid is introduced to assist memory.

No one "naturally" remembers that the correct responses to "5 + 7 = ___" is "12." Methods selected to "mediate" vary from primitive and cumbersome to sophisticated and efficient. Child A arranges 5 objects and 7 objects in piles, then counts them as a single collection from 1 to 12. Child B also arranges piles of objects, but starts at 5 and counts on using fingers. Child C starts from the bigger number (7) and uses fingers to count 5 more.

Child D calls up a mental image of 7 counters in a ten-frame and 5 more



and notices that 3 of the 5 would fill up the ten-frame leaving 2 . . . . and that's 12. Child E argues "7 is 5 and 2 more, so two 5's are 10 and 2 more makes 12." Child F notices that if one of the 7 is switched to the 5, that would make 6 and 6 or 12. Child G has said the sentence "five + seven = twelve" and heard it said and written the complete sentence so often that reading "five + seven = - - - -" leads to "twelve" rolling of the tongue without further conscious thought. Child H has used such methods so often the "mediation" fades out: "5 + 7" is seen as a symbol for "12" . . . and has finally achieved the directness of "natural memory."

The eight specific instances of mediation described above - and many variations not considered - could be ranked according to relative practicality. However, they can be placed more usefully in 2 main categories - (a) those that depend on action outside the mind, and (b) those that are carried out completely within the mind using visual images and inner speech and symbols - at speeds that would lead one observer to characterize them all as "automatic recall."

For practical purposes, all methods carried out in the mind enable children to move ahead with fluency and confidence. Without adult guidance and assistance, many children will keep on "drilling and practicing" crude methods of mediation. In fact, about half of the children spend the first 3 or 4 years counting on their fingers, making hash-marks, counting dots, or some other method that does not utilize the speeds their minds make available. They become "I don't get its" as the flow of arithmetic passes them up.

A simple strategy for helping children develop memory of basic facts seems quite clear:

- be sure all children have some reliable concrete or pictorial method for finding correct responses to such questions as 5 + 7 = ___.

- provide all children with many opportunities to build useful mental images and ways of talking to themselves to reach from what they know to what they don't know.

- persistently focus on the complete sentences that combine the question and the answer - as a line of poetry or a TV commercial. Saying that full sentence over and over again, hearing it and writing it until once begun, the complete sentence rolls off the tongue without further conscious thought.

- let "automatic recall," the final step, happen if and when it will - finally seeing "7 + 5" and "12" as marks that represent the same number.

In the past, our preoccupation with "right answers" has distracted us from the central issue - how are children getting those answers? Consequently, we have left the development of more efficient methods of mediation to chance . . . . ignoring the fact that many children are struggling with primitive approaches and falling further behind. "More drill and practice" without regard for precisely what it is children are doing, is a destructive simplification of a serious problem. Helping children learn efficient ways "to tie strings on their fingers" - to help them select useful ways to assist the development of mediated memory - is a challenge to concerned adults.

CONCLUSION

The standard elementary school math curriculum has taken on the image of standardized achievement tests and has become sterile as it tacitly accepts the limits within which testmakers are compelled to work. That is the tyranny of tests!

The results in terms of test scores are a matter of public record . . . . and they are hard to explain away. The reaction of the mass media, legislators and the public are increasingly restless and becoming less friendly. The challenge to educational leadership is clear and insistent . . . . to turn away from the tyranny of tests and to begin creating a climate designed to help all children develop their demonstrated capacities to learn.

CDA MATH is the program Robert Wirtz has developed as an alternative to the standard elementary school mathematics curriculum. It consists of:

PUPIL MATERIALS

--- Patterns and Problems, Levels A-F
--- Individualized Computation, Levels a_1-f_2
--- My Progress Book, Levels abc, def

TEACHER MATERIALS

--- Drill and Practice at the Problem Solving Level
--- Banking on Problem Solving
--- Common Sense in Arithmetic: A Language Experience/ Language Development Approach to Computational Literacy

PARENT MATERIALS

--- Making Friends with Numbers, addition and subtraction
--- Making Friends with Numbers, multiplication and division
--- "My Full Load of + and - Facts"
--- "My Full Load of x and ÷ Facts"

In the CDA MATH program, students work with manipulative materials, sketches (or other representational drawings), and numbers as they solve problems. At times, problems are presented to them; at other times, they pursue questions of their own design. Always the intent is two-fold: to increase the store of experiences that they can remember and recall with ease, and simultaneously to develop reasoning abilities.

The structure of the program and sequencing within it are integrally related to this approach to learning. When encountering a new idea, students experiment with materials that they can move about. They keep records of the experiments they perform, thereby linking the arithmetic notation with a physical model.

In the books for youngest learners and slow performers, level a, experiments for addition, subtraction, multiplication and division are introduced using experiment forms and concrete objects that can be placed in the forms. In the level b books, the same operations are developed; sketches replace the physical experiments, though always children are encouraged to use objects that they can count if the sketches become confusing. In level c and d books, sketches are accompanied by the arithmetic notation until, gradually, the notation appears without the drawing.

Thus, there is a movement from manipulative to representational to abstract activities in levels a, b, c, and d. At levels e and f the operations on long multiplication and division are introduced, as well as work with fractions. Again, work with physical materials is recommended for some children; the books contain the representational models so that students can build visual images to understand the algorithms.

Practice in the facts is presented in problem settings, frequently with puzzle-like appeal. As they work on problems, students try out one solution and then another; they refine their guesses and formulate new hypotheses; they organize data and form generalizations. It is characteristic of most of these problems that they can be pursued on a variety of levels; if work with abstract symbols for numbers becomes confusing, sketches or movable objects make the problem clearer.

Language Experience and Language Development

Language experience is an essential ingredient of CDA Math: vocabulary grows from the experiences children have. As they carry out activities, they talk with each other describing what they observe and what they are doing. The teacher introduces new vocabulary as it becomes meaningful in the context of the activity. Students are not burdened with such labels as "addends" or "denominators," "distributive" or "commutative properties," until the words become useful to describe an observation or to extend understanding. Development of language is rooted in experience.

The pages in the student booklets contain few words. Youngsters who have difficulty reading need not be held back in their mathematical understandings. Yet, this format with few printed words, promotes oral language development. Students must examine the pages to understand their content; they find the words to talk with each other about their observations and the teacher introduces vocabulary that helps them describe what they see, as well as the language needed to express relationships.

Implementation of the Program

Before introducing materials to students, teachers attend workshops where the approach to learning underlying the program is introduced and exemplified through activities and examination of the material. Teachers become familiar with the student booklets at the various levels and with the other resources so that they can select forms and activities for extension of the program and placement.

As teachers introduce the curriculum in their classrooms, it is important that their efforts be supported from within the school system. CDA stresses the role of principals and resource teachers as curriculum leaders within their schools. Special in-service sessions are arranged to develop leadership roles to support the program.

Parent and Community Involvement

Incorporated into the CDA program are avenues of communication between home and school. Specific materials are prepared to make it possible for parents to support and assist their children with instructional materials related to the school curriculum.

So that children may share their school work with parents, the pages of the Individualized Computation books are perforated. Frequently, teachers staple together a unit of work, after a child has completed the progress test, and send the unit home. At levels a, b, and c, there are letters to parents on the backs of the pages (letters available in Spanish) explaining the purpose of the section and suggesting activities that can be pursued at home.

In addition, kits of materials entitled Making Friends With Numbers have been prepared for parents to help children develop an ease of telling about and remembering the basic facts of addition, subtraction, multiplication, and division. Containing counting frames, fact charts, read aloud cards, and guides, these kits enable parents to provide drill and practice in an enjoyable way, giving the child the individual attention that a parent can provide.